Bid-Ask Spread Dynamics

- A Market Microstructure Invariance Approach

Master Thesis

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Abstract

The theory of Market Microstructure Invariance proposed by Kyle and Obizhaeva (2010) is presented and tested on spread data for bond futures. The data used are transformations from over 150,000 observations of futures on German government debt securities (Schatz and Bund) and 10-year US treasury notes. To account for the possible presence of long memory processes, we perform a GPH-test that displays no long memory processes in the data. Our findings support the theory of Market Microstructure Invariance, however the results differ between futures.

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INTRODUCTION

When acting in the financial markets, a trader can choose to submit limit orders or pay the cost for immediate trading and cross the bid-ask spread. The price of immediate trading – the size of the bid-ask spread - signifies the cost for market participants to constantly be guaranteed a counterpart for trades. Often used as a measure of liquidity, the bid-ask spread is of utter importance for traders´ strategies concerning market or limit orders since it reflects a transaction cost (Harris and Hasbrouck, 1996). Hence, understanding the dynamics of the bid-ask spread is essential for everyone participating in the financial markets.

The subject of the components and the predictability of the bid-ask spread have been thoroughly addressed within the fields of market microstructure and game theory. Foucault (1999) and Foucault et al. (2005) use game-theoretical dynamic models to advocate the bid-ask spread as fundamental for trading strategies. Within the framework of market microstructure, the models employ inventory liquidation, information asymmetry or transaction costs to explain the spread. Stoll (1989), Huang and Stoll (1997), and Bollen, Smith and Whaley (2004) investigates the bid-ask spread by computing inventory holding costs, i.e. the cost for the market maker to supply liquidity. Information asymmetry, the approach that the bid-ask spread is a product of differences in the levels of information between the specialist and the customer, was first introduced by Bagehot (1971) and further developed by Glosten and Milgrom (1985). In the model, the uninformed market maker, knowing that an order might be information-motivated, revises his or her expectations for the asset when the order is received. The expectations are then incorporated by the market maker in the quoted bid and ask prices.

Roll (1984), being the first, derived an estimator to estimate the bid-ask spread in the equity market. Roll acknowledged that the quoted spread not always corresponds to the effective spread. Thus, the quoted spread sometimes exaggerates the real transaction costs faced by traders. Under the prerequisite that the market is efficient, Roll uses the relationship between transaction price changes to estimate the bid-ask spread indirectly. This approach has later been extended by Stoll (1989) and George et al. (1991) with more complex estimators. However, these estimators have received criticism for having problem estimating quotes and price change covariance (Chen and Blenman, 2003). An extended model of serial covariance bid-ask spreads, which circumvented the shortcomings of Stoll (1989), was introduced in 2003 by Chen and Blenman.

A limitation to some of these aforementioned models is that they make explicit assumptions concerning the dynamics of the market. Either by addressing inventory costs or information asymmetry as more determinative. Common for prior models and estimators is also that they do not attend to the reactions on the bid-ask spread by trades and how the impact is correlated with the volume of the trade.

In 2010, Kyle and Obizhaeva approached the subject of market microstructure and the bid-ask spread in a somewhat new manner and introduced a theory of Market Microstructure Invariance. The theory, consistent with traditional theories using information asymmetry and inventory holding costs, build on the intuition that stocks with high and low levels of trading activity differ in the rate at which the time clock generating trading activity ticks. The predictions of the theory involve market impact, order size changes and effects on the bid-ask spread caused by changes in trading activity. It predicts that a one percent increase in trading activity decreases the bid-ask spread by one third of one percent. The theory was tested on portfolio transition data and the results corresponded to a large extent with the theoretical values.

In addition to the effects on the bid-ask spread by changes in trading activity, the behaviour of the bid-ask spread is also to some extent explained by Groß-Klußmann and Hautsch (2011). They found long-range dependency in the bid-ask spread, by applying a Geweke and Porter-Hudak test (1983), when developing a model for forecasting.

The presence of a long memory effect is not unusual in financial market data. During the last twenty years, extensive research has been conducted on long memory modelling within the fields of macro and financial studies (Bhardwaj and Swanson, 2006). The areas investigated include volatility data (Ding and Granger (1996) and Andersen et al. (2003)), traded volumes (Lux and Kaizoji, 2007) and trade durations Deo et al. (2010). The incidence of a long memory effect might cause problems with specification if not taken into consideration when modelling.

Our study contributes to the existing literature by testing the theory of Market Microstructure Invariance (Kyle, Obizhaeva, 2010) on market data for bond futures to examine if the findings from the stock market regarding the bid-ask spread are applicable on the derivatives market. In contrast to Kyle and Obizhaeva who used time periods of one day, we have examined intraday trading by employing time periods of 30 minutes. To account for the findings of Groß-Klußmann and Hautsch (2011) we examine our data for long-memory dependence.

We find that the presence of long memory dependence should most likely be discarded, in contrast to the findings of Groß-Klußmann and Hautsch (2011). Further, we show that a one percent increase in trading activity causes the bid-ask spread to decrease closely to one third of one percent. High adjusted R-squared values indicate that the estimated model fits the data well.

The remainder of the paper is organised as follows; chapter 2 describes the theory of Market Microstructure Invariance, chapter 3 explains long memory, in chapter 4 we present our data, chapter 5 derives our empirical test models, chapter 6 presents our results and chapter 7 concludes the paper and some last concerns.

MARKET MICROSTRUCTURE INVARIANCE MODEL

Kyle and Obizhaeva (2010) have developed a model to explain the dynamics of trading. They consider the difference in activity between markets as a product of difference in time speed. As an illustrative example they give chess. The game of chess has certain rules and tactics; these do not change if the game is played with a clock. In the same sense the rules of trading does not change if the time speed is increased. The amount of "bets" (trades) increase but the fundamentals behind trading does not change.

The order-flow, which consists of "bets", follows a Poisson process. Kyle and Obizhaeva argue that the risk transferred by traded shares is more meaningful than the traded volume; hence the traded risk is the measure used in the model. "Bet"-risk[1] is defined as bet value (share price times share quantity) times the volatility (standard deviations per day). Further "trading activity"[2] is defined as the arrival rate of bets times the value of bets and the volatility.

Kyle and Obizhaeva apply two irrelevance principles; "Modigliani-Miller irrelevance" and "Time-clock irrelevance". The "Modigliani-Miller irrelevance" means that bet risk and bet arrival rate are unaffected by splits etc. In the same manner the "Time-clock irrelevance" implies that changing the time-speed by speeding up or slowing down the time does not change the rules of the trading game. By changing the time-speed the trading activity increases by an increase in the number of bets and their risk (by increasing volatility).

The theoretical aspects of the irrelevancies do not imply that all securities act as one. To be able to test this on market data, assumptions has to be applied. The assumptions allows for generalisations between different securities in different markets. This method enables for comparison of behaviour as a result of difference in time-speed. Kyle and Obizhaeva make the following assumptions:

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[1] $B=P*Q*\sigma$

[2] $W=\gamma*P*Q*\sigma$

- Trading Game Invariance: The probability of the trading game invariant (defined as ration of bet size to the square root of bet arrival rate) is the same across stocks and across time for the same stock.

- Market Impact Invariance: For all stocks, the same constant fraction ψ of price volatility results from the linear price impact of bets.

- Bid-Ask Spread Invariance: For all stocks, the expected bid-ask spread cost of a bet is the same fraction ϕ of market impact cost.

Kyle and Obizhaeva name these assumptions “market microstructure invariance” and the variables “market microstructure invariants”. These ideas lead to the following implications:

- Trading Game Invariance implies that if trading activity is observed to increase by one percent, then the increase in trading activity resulted from an increase in the arrival rate of bets by two-thirds of one percent and an increase in bet size of one-third of one percent. Trading game invariance implies that the shape of the distribution of bet size does not change as the level of trading activity changes.

- Market Impact Invariance together with trading game invariance implies that increasing trading activity by one percent increases the market impact cost of trading one percent of average daily volume by one-third of one percent, when trading costs are measured in basis points per dollar traded, holding volatility constant.

- Bid-ask Spread Invariance together with Trading Game Invariance implies that increasing trading activity by one percent decreases the bid-ask spread by one third of one percent, when costs are measured in basis points per dollar traded, holding volatility constant.

This results in an increase in trading activity when the time clock is moving faster. The activity increases because the arrival rate of bets and the variance increases. The variance increases proportionally with the arrival rate of bets. Since price volatility is the standard deviation of returns (square root of variance), volatility increases half as fast as variance. This means that as trading activity increases, the arrival rate of bets increases twice as fast as volatility.[3]

Define the time-speed as H and benchmark it at 1, by speeding up the time H < 1 and by slowing down H > 1. With a lower H trading activity increases for two reasons; the number of bets (proportionately with 1/H) and the volatility increases ($1/H^{1/2}$, since volatility is the square root of variance). Therefore the trading activity increases proportionately with $1/H^{3/2}$.

Since the theory does not make any assumptions about how market participants act (except that their behaviour is the same for all markets), the model is not in conflict with adverse selection or inventory models.

MATHEMATICS TO SPREAD PREDICTIONS

Bet risk ($\tilde{B}$) and trading activity (W) are defined as;

$$\tilde{B} = \tilde{Q} \times P \times \sigma \quad (1)$$

$$V = \gamma \times E\left(\left|\tilde{Q}\right|\right) \quad (2)$$

$$W = V \times P \times \sigma \quad (3)$$

[3] If the volatility increases by one percent (1+0.01), the variance will increase by $(1+0.01)^2$. Consider that the volatility increases by one percent in 2 periods, then the variance will have the same effect after one period, in other words increases twice as fast.

where $\tilde{Q}$ is shares per bet, P is price, γ is bet arrival rate, σ is volatility and V is volume. The time length of each period is one day of trading.

Algebraically we can express the effects of Modigliani-Miller irrelevance and time-clock irrelevance, where;

γ = Bet arrival rate

σ = Volatility

η = Split factor

δ = Debt financed proportionate dividend

H = Time-clock speed (benchmarked at 1)

* = Benchmark values

$$\gamma = \frac{1}{H} \times \gamma^* \tag{4}$$

$$\tilde{Q} = \eta \times \tilde{Q}^* \tag{5}$$

$$P = \frac{(1-\delta)}{\eta} \times P^* \tag{6}$$

$$\sigma = \frac{1}{H^{1/2} \times (1-\delta)} \times \sigma^* \tag{7}$$

The Modigliani-Miller irrelevant transformation η and δ do not affect the bet risk or the trading activity. However, the time-clock changes do.

$$\tilde{B} = \tilde{B}^* \times H^{-1/2} \tag{8}$$

$$W = W^* \times H^{-3/2} \tag{9}$$

H is a variable that we cannot observe, but we can easily solve H from W.

$$H = \left(\frac{W}{W^*} \right)^{-2/3} \tag{10}$$

By using the expression for H we can state the changes in bet risk and bet arrival rate as a function of trading activity.

$$\tilde{B} = \tilde{B}^* \times \left(\frac{W}{W^*} \right)^{1/3} \tag{11}$$

$$\gamma = \gamma^* \times \left(\frac{W}{W^*} \right)^{2/3} \tag{12}$$

This implies that as trading activity increases, both the arrival rate of bets and the bet risk increases.

Kyle and Obizhaeva define $\tilde{I}$ as the "trading game invariant", which is the ratio of bet size to the square root of the bet arrival rate:

$$\tilde{I} = \frac{\tilde{B}}{\gamma^{1/2}} \tag{13}$$

By using the product of equations 5, 6 and 7 divided by the square root of equation 3 we express the "trading game invariant". The Modigliani-Miller irrelevance and time clock irrelevance coefficients all cancel out. This gives us:

$$\tilde{I} = \frac{\tilde{Q} \times P \times \sigma}{\gamma^{1/2}} = \frac{\tilde{Q}^* \times P^* \times \sigma^*}{\gamma^{*1/2}} \tag{14}$$

Kyle and Obizhaeva refer to $\tilde{I}$ as the trading game invariant. The intuition is that the invariant is not affected by irrelevant alterations, such as the Modigliani Miller and time clock transformations. Kyle and Obizhaevas model states that the risk of a bet can be measured invariant to the time speed.

$\tilde{I}$, $\tilde{Q}$, and $\tilde{B}$ have the same shape, but different scaling.

$$\frac{\tilde{I}}{E\left\{\left|\tilde{I}\right|\right\}} = \frac{\tilde{Q}}{E\left\{\left|\tilde{Q}\right|\right\}} = \frac{\tilde{B}}{E\left\{\left|\tilde{B}\right|\right\}} \tag{15}$$

By using equation 1 and 3 we get the following.

$$W = \gamma \times E\left\{\left|\tilde{B}\right|\right\} \tag{16}$$

Solving for γ and $E\left\{\left|\tilde{B}\right|\right\}$ by using equation 16 and 13.

$$\gamma = E\left\{\left|\tilde{I}\right|\right\}^{-2/3} \times W^{2/3} \tag{17}$$

$$E\left\{\left|\tilde{B}\right|\right\} = E\left\{\left|\tilde{I}\right|\right\}^{2/3} \times W^{1/3} \tag{18}$$

These equations state that changes in trading activity leads to bet frequency increasing twice as fast as bet risk. The difference to equation 11 and 12 is that the trading game invariant is used in contrast to benchmark levels.

Transaction Costs

Kyle and Obizhaeva assume that the costs of trading consist of two parts, permanent linear price impact and transitory bid-ask spread. C_L denotes the price impact cost, C_K the bid-ask spread cost and λ is the price impact of trading one share (measured in units of dollar per share squared) (Kyle, 1985). Assuming that the order is executed by "walking the book"[4], the expression is multiplied by one half. The expected price impact cost is a linear slope:

$$C_L = \frac{1}{2} \times \lambda \times E\left\{\tilde{Q}^2\right\} \tag{19}$$

[4] As the price impact is a linear function, for every transaction we move further along the price impact line. To calculate the transaction cost, the total market impact is divided by two and applied for all transactions.

κ is the bid ask spread for one share in dollars per share.

$$C_K = \kappa \times E\left\{\left|\tilde{Q}\right|\right\} \tag{20}$$

In line with the Modigliani-Miller irrelevance and time clock irrelevance it is assumed that splits, leverage dividends or time speed do not affect the cost. A stock split has proportionate effects on the number of shares traded, while leverage and time clock changes do not.

$$\lambda = \lambda^* / \eta^2 \tag{21}$$

$$\kappa = \kappa^* / \eta \tag{22}$$

To make price impact and its cost endogenous, the price impact is assumed to lead to a fraction ψ^2 of price variance. This enables the price volatility to be explained by both announcement effects and trading effects. Daily price impact can therefore be expressed as product of price variance and ψ^2, or the arrival rate of bets times its price impact.

$$\psi^2 \times \sigma^2 \times P^2 = \gamma \times \lambda^2 \times E\left\{\tilde{Q}^2\right\} \tag{23}$$

Rewriting the equation for λ

$$\lambda = \frac{\psi \times \sigma \times P}{\gamma^{1/2} \times E\left\{\tilde{Q}^2\right\}^{1/2}} \tag{24}$$

By using equation 24 in 19 and the definition of $\tilde{B}$ in 1, we get the following.

$$C_L = \frac{1}{2} \times \psi \times \frac{E\left\{\tilde{B}^2\right\}^{1/2}}{\gamma^{1/2}} \tag{25}$$

The equation expresses a linear relationship for market impact cost of bet risk and bet arrival rate. However the linear approach is the most commonly used. (Chen, Stanzl and Watanabe, 2002)

Kyle and Obizhaeva make the following assumption to get a stable endogenous relationship between the bid-ask spread and the expected spread costs:

- The expected bid-ask spread cost of a bet is a constant fraction ϕ of the expected market impact cost of a bet.

$$C_K = \phi \times C_L \tag{26}$$

By using equations 26 and 20 we get the following expression for the bid-ask spread.

$$\kappa = \frac{\phi \times C_L}{E\left\{\left|\tilde{Q}\right|\right\}} \tag{27}$$

TIME CLOCK IRRELEVANCE AND TRADING COSTS

The expression for the "trading game invariant" (13) can be combined with equation 25 to express the market impact cost as a function of moments of the invariant.

$$C_L = \frac{1}{2} \times \psi \times E\left\{\tilde{I}^2\right\}^{1/2} \tag{28}$$

The expected market impact cost is not defined by trading activity W. This is in line with the idea that the speed of the trading game does not change the "fundamentals" of the game.

The market impact of trading one entire day's expected trading volume V, expressed as a fraction of one day's price volatility σP is given by:

$$\frac{\lambda \times V}{\sigma \times P} = \psi \times \frac{E\left\{\left|\tilde{Q}\right|\right\}}{E\left\{Q^2\right\}^{1/2}} \times \gamma^{1/2} \tag{29}$$

This can also be expressed by using moments of the invariant and trading activity by using equation 15 and 17.

$$\frac{\lambda \times V}{\sigma \times P} = \psi \times \frac{E\left\{\left|\tilde{I}\right|\right\}^{2/3}}{E\left\{\tilde{I}^{2}\right\}^{1/2}} \times W^{1/3} \tag{30}$$

With the expressions for bet risk (1) and bid-ask spread (27), the bid-ask spread as a fraction of one day's dollar volatility (σP) can be expressed:

$$\frac{\kappa}{\sigma \times P} = \frac{\phi \times C_L}{E\left\{\left|\tilde{B}\right|\right\}} \tag{31}$$

Plugging in equation 28 and 18, we get an expression that only depends on moments of the trading game invariant and trading activity.

$$\frac{\kappa}{\sigma \times P} = \frac{1}{2} \times \phi \times \psi \times \frac{E\left\{\tilde{I}^{2}\right\}^{1/2}}{E\left\{\left|\tilde{I}\right|\right\}^{2/3}} \times W^{-1/3} \tag{32}$$

Equation 30 and 32 states that a bet representing one percent of daily trading (holding price and volatility constant) leads to one third of a percent increase in impact cost and one-third of a percent decrease in spread cost.

To further explain the concept, equation 32 can be transformed:

$$\left(\frac{\kappa}{\sigma \times P}\right) \times W^{1/3} = \frac{1}{2} \times \phi \times \psi \times \frac{E\left\{\tilde{I}^{2}\right\}^{1/2}}{E\left\{\left|\tilde{I}\right|\right\}^{2/3}} \tag{33}$$

The equation above express the spread as a fraction of price volatility times one divided by the trading activity raised to minus one third. Since the right hand side only depends on invariants, a positive change in trading activity must lead to a decrease in the spread as a fraction of price volatility.

If trading activity increases by one percent, the number of bets represented by one day's trading volume increases by two-thirds of one percent according to equation 16. The impact cost will increase by one-third of one percent (equation 30). The bid-ask spread is reduced by one-third of one percent because the spread is scaled by volatility that increases by one-third of one percent (equation 32).

As an empirical hypothesis

The above theoretical results do not assume that the invariant distribution is the same for different stocks or different time periods. Therefore they are not empirically testable. To be able to investigate the findings Kyle and Obizhaeva make testable assumptions about the properties of different stocks:

- Trading Game Invariance: For all stocks, the distribution of the trading game invariant $\tilde{I}$ is the same.
- Market Impact Invariance: For all stocks, the linear price impact of bets explains a fraction ψ of price volatility which is constant across stocks.
- Bid-Ask Spread Invariance: For all stocks, the bid-ask spread is the same fraction ϕ of the expected market impact cost of a bet.

These hypotheses are referred to as the "Market Microstructure Invariance". The basic idea which forms the theory is; for all stocks the trading game is the same, except for some irrelevant transformations (Modigliani-Miller and time-clock transformations).

The trading game invariance and the market impact invariance imply that the expected market impact cost of a bet is the same for all stocks. Paired with the bid-ask spread invariance, it implies that the expected bid-ask spread cost of a bet is the same for all stocks. Kyle and Obizhaeva test this by investigating the effects of trading a given fraction of average daily volume.

The theory states that if two stocks have the same level of trading activity, the impact costs and spread costs of trading the same percentage will be the same. If the trading activity differ so will the transaction costs as shown in equation 30 and 32.

Equation 33 implies that the bid-ask spread, scaled by volatility and price, satisfies:

$$\frac{\kappa}{\sigma \times P} = \frac{\kappa^*}{\sigma^* \times P^*} \times \left(\frac{W}{W^*}\right)^{-1/3} \qquad (34)$$

An increase in trading activity decreases the spread scaled by volatility and price one-third as fast.

Kyle and Obizhaeva performed a study on portfolio transition[5] data, which consisted of over 400,000 stock-trades on NYSE[6] and NASDAQ. The results of the study are that the spread decreases somewhat faster then what the theory predicts when trading activity increases. Their estimate of the power, which the trading activity is raised to, is -0.39 with a standard error of 0.025. However, estimated values for each series varies from -0.19 to -0.46.[7]

5 Portfolio transition: When a portfolio is sold and bought again for the purpose of moving it between two asset managers for example.

6 New York Stock Exchange

7 For further reading please see Kyle and Obizhaeva, 2010

LONG MEMORY

Groß-Klußmann and Hautsch (2011) forecasted bid-ask spreads by using an autoregressive conditional Poisson (ACP) model with a long memory extension. The results showed that their model outperformed other models in forecasting, and had the possibility, when implemented in a simple algorithmic trading model, to lower transaction costs by up to 13%.

For the purpose of this paper, the relevant part is that they showed that there are low frequencies within the bid-ask spread time-series that caused a long memory effect.

Long memory is defined as dependence between observations widely separated in time. In contrast "short-memory" processes, for example modelled with ARMA-processes, decays rapidly as the lags increases. One way to model long memory is by using an ARFIMA model (AutoRegressive Fractionally Integrated Moving Average, (p,d,q)). It allows for differentiation of non-integer values. Diebold and Rudebusch (1989) showed that allowing for non-integer values of *d* provides "parsimonious yet flexible modelling of low-frequency variation". (Diebold and Rudebusch, 1989)

Geweke and Porter-Hudak (1983) proposed a two-step procedure to estimate long memory dependence. Diebold and Rudebusch (1989) argue that it has more benefits than competing models such as Maximum Likelihood, due to the risk of misspecification.

An ARFIMA model is the same as ARIMA with the difference of allowing for non-integer values of *d*. To model it, the first step is to estimate the order of integration, secondly the series is transformed and the ARMA process is fitted to the new series.

Lets denote $X_t = (1 - L) Y_t$, an ARIMA process can then be expressed as

$$\Phi(L)X_t = \Theta(L)\varepsilon_t \qquad (35)$$

To allow for fractional differentiation we add a lag operator raised to $\hat{d}$.

$$(1-L)^{\hat{d}} X_t = \Phi^{-1}(L)\Theta(L)\varepsilon_t \equiv u_t \quad (36)$$

As $\hat{d}$ equals 1 + *d* of the Y series, a value of *d* equal to zero corresponds to a unit root in Y_t.

FIRST STEP – ESTIMATE $\hat{d}$

The estimation of $\hat{d}$ relies on the spectral density[8] of X_t. Through a spectral regression the estimate can be obtained. (Groß-Klußmann and Hautsch, 2011)

$$\ln\{I(\lambda_j)\} = \beta_0 + \beta_1 \ln\{4\sin^2(\lambda_j/2)\} + \eta_j, \qquad j = 1, 2, \ldots, K \quad (37)$$

The periodogram $I(\lambda_j)$ is given by the fourier frequencies of X_t. (Shumway and Stoffer, 2006) The estimate of β_1 represents $\hat{d}$. (Groß-Klußmann and Hautsch, 2011)

The variance of β_1 is given by the OLS estimator, and the theoretical variance of η is equal to $\pi^2/6$. By imposing the theoretical variance the efficiency increases. (Diebold and Rudebusch, 1989)

The β_1 parameter estimates the order of integration for the series, where the order of integration is 1+ $\hat{d}$.

[8] "… any stationary time series may be thought of, approximately, as the random superposition of sines and cosines oscillating at various frequencies." (Shumway and Stoffer, 2006)

SECOND STEP - MODELLING LONG MEMORY

The X_t series is then differentiated of the order *d*, and modelled as an ARMA (p, q) process. Since the estimates from the periodogram regression are consistent, the estimates in the second step will also be consistent. (Diebold and Rudebusch, 1989)

$$\Phi(L)(1-L)^d X_t = \Theta(L)\varepsilon_t, \qquad \varepsilon_t \sim (0,\sigma_\varepsilon^2) \qquad (38)$$

The above is an ARFIMA model, where $\Phi(L)=1-\phi_1 L-\ldots-\phi_p L^p$, $\Theta(L)=1-\theta_1 L-\ldots-\theta_q L^q$. All roots of $\Phi(L)$ and $\Theta(L)$ lie outside the unit circle and *d* is allowed to assume values in the real set of numbers. (Diebold and Rudebusch, 1989)

If the model is transformed we get the following

$$X_t = (1-L)^{-d} A(L)\varepsilon_t, \qquad \varepsilon_t \sim (0,\sigma_\varepsilon^2) \qquad (39)$$

where A(L) is $\Theta(L)/\Phi(L)$. $(1-L)^{-d}$ is calculated by using a binomial expansion.[9]

[9] For further reading see Diebold and Rudebusch (1989) and Geweke and Porter-Hudak (1983)

DATA

For the purpose of testing the theory, market data for three futures were used. From the data, the quoted spread, value and volume for the traded futures as well as bid and ask was extracted with a frequency of every 60-second. The data was retrieved from Bloomberg.

Table 1.*

Future	Underlying asset	Nominal contract value	Remaining lifetime of the deliverable bonds	Settled on
Euro Bund Future	German government debt securities	100 000 EUR	8 ½ to 10 ½ years	Eurex
Euro Schatz Future	German government debt securities	100 000 EUR	1 ¾ to 2 ¼ years	Eurex
10-Year U.S. Treasury Note Future	10-Year U.S. Treasury Note	100 000 USD	6 ½ to 10 years	CBOT

*Source: Bloomberg Database, CME Group and Fixed Income Trading Strategies (Eurex)

In contrast to Kyle and Obizhaeva, who tested their model for stock data, the data used in this paper are for fixed income futures. The three futures used are the Euro Bund Future, Euro Schatz Future and 10-Year U.S. Treasury Note Future. Bund and Schatz settle on Eurex and have German government debt securities as underlying instrument. The 10-Year U.S. Treasury Note Future settle on CBOT[10]. Trading activity differs between the investigated futures with the 10-Year U.S. Treasury Note being the most frequently traded and Schatz the least.

[10] Chicago Board of Trade

Table 2.*

Future	Time period	Observations	Transformed observations
Euro Bund Future	2011-03-16 08:00-2011-06-08 12:29	47 065	1 631
Euro Schatz Future	2011-03-16 08:00-2011-06-0810:29	41 981	1 629
10-Year U.S. Treasury Note Future	2011-03-16 06:00-2011-05-3013:29	63 510	2 349

*Source: Bloomberg Database

In order to implement an accurate analysis the dataset was processed before included in the regression. An average price of the security was calculated for every minute when trading was carried through by dividing the value with the volume of the completed trades. From these prices an average price for every 30-minute period was calculated. Same procedure was conducted for the bid and ask prices in order to attain an average bid-ask spread per every 30 minute. This was necessary to account for the minutes when no trades were carried through. The volatility was computed through the standard deviation of the average minute prices for every 30-minute period. Using these prices, trading activity (W) for every 30 minute was calculated.

EMPIRICS

Kyle and Obizhaeva formulate the spread with the following equation[11].

$$\left(\frac{\kappa_t}{\sigma_t \times P_t}\right) = \left(\frac{\kappa^*}{\sigma^* \times P^*}\right) * \left(\frac{W_t}{W^*}\right)^{\alpha} \tag{40}$$

However, it is not necessary to express this with benchmark values. By changing the benchmark values to the values from the last period the equation will denote the change as a function of the change in trading activity.

$$\left(\frac{\kappa_t}{\sigma_t \times P_t}\right) = \left(\frac{\kappa_{t-1}}{\sigma_{t-1} \times P_{t-1}}\right) * \left(\frac{W_t}{W_{t-1}}\right)^{\alpha} \tag{41}$$

By transforming the equation to represent the change instead does not affect the fundamental idea behind the theory. If the level of trading activity explains the size of the spread against benchmark values, the difference in trading activity between two periods can explain the difference in the spread as well.

By taking the logarithms of equation 41 we get the following expression.

$$\ln\left(\frac{\kappa_t}{\sigma_t \times P_t}\right) = \ln\left(\frac{\kappa_{t-1}}{\sigma_{t-1} \times P_{t-1}}\right) + \alpha \times \ln\left(\frac{W_t}{W_{t-1}}\right) \tag{42}$$

or

$$(1-L)\ln\left(\frac{\kappa_t}{\sigma_t \times P_t}\right) = \alpha \times \ln\left(\frac{W_t}{W_{t-1}}\right) \tag{43}$$

The integrated process is explained by the difference in trading activity.

[11] It should be noted that the equation do not specify what happens to the spread when there is no trading activity. For Kyle and Obizhaeva this is not a problem in most scenarios because their time period is one trading day.

DIFFERENCE IN ESTIMATION

Kyle and Obizhaeva approach the empirical modelling in a different manner. Instead of testing the spread, they formulate an equation for the total trading cost (price impact plus spread cost). They also argue that there are difficulties in estimating the predictions. We deviate from this reasoning when modelling the spread predictions. For price impact, there are difficulties because a standard database does not reveal insight to the single bets. However, for spread predictions this is not necessary, changes to the spread does not depend on what type of bet it is (short or long position). This insight results in the possibility to estimate the spread as above (43) since all variable are in real terms. This enables to release the assumption that different assets have the same distribution, since we do not estimate cross-sectional dependence. The derivatives are though assumed to have the same distribution for all time periods.

COMBINING WITH LONG MEMORY

If the GPH-test results indicate that the series contains long memory dependence it can be modelled as above.

$$X_t = (1-L)^{-d} A(L)\varepsilon_t, \qquad \varepsilon_t \sim (0,\sigma_t^2) \qquad (44)$$

In this case the X series represents the differentiated logarithm of our spread as a fraction of the price volatility.

$$(1-L)\ln\left(\frac{\kappa_t}{\sigma_t \times P_t}\right) = (1-L)^{-d} A(L)\varepsilon_t, \qquad \varepsilon_t \sim (0,\sigma_t^2) \qquad (45)$$

This states that the first difference can be explained by the fractionally differentiated epsilon with an ARMA process.

If the series contains long memory, the spread will not only be a function of changes in the trading activity. Therefore it is necessary to combine equation 43 with 44. Hence the model we want to estimate will therefore be

$$(1-L)\ln\left(\frac{\kappa_t}{\sigma_t \times P_t}\right) = \alpha \ln\left(\frac{W_t}{W_{t-1}}\right) + (1-L)^{-d} A(L)\varepsilon_t, \quad \varepsilon_t \sim (0, \sigma_t^2) \quad (46)$$

This model has the ability to explain the difference in the spread as a function of trading activity and the long memory process (low frequency waves).

If the GPH-test indicates that there is no long memory process in the data, the modelling will be conducted without consideration to the long memory part (ARIMA) and in line with Kyle and Obizhaeva.

If the long memory process is modelled it will not affect the conclusion from the parameter value of alpha. The reasoning behind this is the following; consider a series that contains long memory. Through dividing both sides of equation 44 with the low frequency waves, a series unaffected by long memory is obtained. Thus, the series according to the equation will depend on white noise. However, the approach of this paper is that the series does not only depend on itself, it depends on the trading activity. This means that the model is misspecified and depends on trading activity as well. By modelling as equation 45, the problem of misspecification is removed and the alpha will solely represent the effect of the trading activity. If modelled without the long memory component, the problem of misspecification will be present.

EMPIRICAL RESULTS

To conclude if the data fits the theoretical value, an empirical study was performed. Presented below are the results. Full diagnostics can be found in appendix.

GPH-TEST

The findings of Groß-Klußmann and Hautsch (2011) suggested that long-memory processes might be present within our data. Due to the risk of misspecification a GPH-test was conducted.[12]

Table 3.*

$\hat{d}$	**Estimate**	**Std. Error**	**T-stat**	**P-value**
$\hat{d}_{\text{Bund}}$	0.007171	0.023101	0.310402	0.7564
$\hat{d}_{\text{Schatz}}$	0.006332	0.028421	0.222794	0.8238
$\hat{d}_{\text{US}}$	-0.010647	0.017109	-0.622313	0.5339

*Estimation of d

The results indicate that we cannot reject the null hypothesis of non-stationarity (I(1)) for the level series. This implies that the long memory part of the model (46) specified above should not be included.

[12] Periodograms for the Fourier frequencies used for the GPH-tests are found in appendix

AR/MA- PROCESSES

The Autoregressive processes might still occur within the series. A graphical analysis is performed on the correlogram[13] of the dependant series. All series indicates that there exist MA-processes since the autocorrelation function dies abruptly after one lag and the partial autocorrelation functions decays slowly. The length of the process is decided through the SBIC[14] due to its restrictive characteristics compared to other information criterions.

Table 4.*

SBIC	**MA(1)**	**MA(2)**	**MA(3)**	**MA(4)**
Bund	0.686113	0.658734	0.643492	Not significant
Schatz	0.796067	0.775770	0.773056	Not significant
US	0.518548	Not significant	Not significant	Not significant

*Estimation of MA-process

ESTIMATION OF ALPHA

When performing diagnostics on our models, heteroskedasticity was found for the US series. To account for this, the model where estimated using Newey-West (HAC).

The results for the estimated parameter are to a great extent in line with the theoretical values. For the future on the 10-year US Treasury note the coefficient

[13] Correlograms are found in appendix

[14] Schwartz bayesian information criterion

coincides with theoretical value of -0.330 with a standard error of 0.007. The model on the Euro Bund future gives a fairly close estimate of -0.266 with standard error of 0.005. However, the estimate for the Euro Schatz future was -0.206 with a standard error of 0.005, which does not support the theoretical value. For further diagnostics Wald tests[15] were performed on all series. The null hypothesis that the theoretical values are correct was rejected for the Bund and Schatz. For the US future the null hypothesis could not be rejected. For all three models, the data fits the model rather accurately with adjusted R^2-values of 0.73 (US), 0.74 (Bund) and 0.69 (Schatz).

COMMENTS

Apart from the Schatz series, the theoretical value of a decrease in the spread by one-third of one percent when the trading activity increase of one percent is accurate for our data. Kyle and Obizhaeva's findings are somewhat supported in this study. They found that the spread decreased somewhat faster when an implementation shortfall[16] regression was used. Our results indicate the opposite for the futures market. Even though the Schatz estimate suggests that the spread only decreases with 0.21 percent, this might be due to some measurement error. It is noticeable that there might be a relationship between the level of trading activity and the response in changes in the bid-ask spread. The futures on US treasury notes are the most frequently traded and the Schatz futures are the least traded.

[15] Results from Wald tests are found in appendix

[16] Shortfall implementation is when the difference between the execution price and the closing price the day before is used to calculate changes.

Conclusion

This paper investigates the theoretical model of Market Microstructure Invariance proposed by Kyle and Obizhaeva (2010). The model assumes that different levels of trading activity results from difference in the time speed. From this assumption, predictions about the bid-ask spread can be derived when examining the trading activity. The model state that the bid-ask spread should decrease by one-third of one percent when the trading activity increase by one percent.

Groß-Klußmann and Hautsch (2011) found that there exist long memory processes within data for the bid-ask spread. This implies dependency between observations widely separated in time.

We tested the model on data for German and US bond futures. Our results from the GPH-test for long memory cannot confirm the findings of Groß-Klußmann and Hautsch (2011). Because of this, long memory was disregarded in the modelling process. However, we found that all series contained MA-processes through a graphical analysis of the correlograms. To decide the accurate length of those processes, the Schwartz Bayesian information criterion was used. When modelled with MA-processes, our results support the theoretical model of Market Microstructure Invariance with alpha values of -0.330, -0.266 and -0.206.

These values differ from the results of Kyle and Obizhaeva. Both are close to the theoretical value, however their results imply that the spread decreases somewhat faster when the trading activity increases. Our results indicate the opposite. This might be due to differences in data and estimation methods. They have made estimations on stock data, which is traded more frequently than the bond futures we examine. There might be a nonlinear relationship between trading activity and the bid-ask spread, which could explain the differences in our results. Another explanation might be the difference in estimation. They use implementation shortfall regressions to estimate the spread. When implementing the regression they assume that the expected movement is zero. If this is estimated during a period that has a trend (bull or bear market for

example), then the results will be biased. We also apply a MA-process to exclude short-term fluctuations.

Although we disregarded the long memory from our estimations, it might still be present. Since we performed a GPH-test on a variable that depended on estimated volatility, the results might be biased.

We have not tested for cross section dependency between the different futures. Therefore we cannot draw any conclusion regarding the hypothesis that all futures are the same.

The results indicate that the model is fairly accurate in predicting changes in the bid-ask spread. With more research in this field it could be useful for market participants when estimating transaction costs and optimising trading strategies. At the moment there are problems using this model for forecasting, since the volatility and future price has to be estimated. This can be discarded by using variables from the last period, but that will most likely lower the power of the forecasts.

Interesting areas for further research are for example:

- For the purpose of so-called high frequency trading to better calculate transaction costs, the theory can be tested on high frequency stock data. Although the problem of short-term fluctuations might be more present.
- Investigate if there is a fixed relationship between the spread, the price and the volatility. Such a relationship would improve the models forecasting possibilities.
- Further research regarding long memory processes within spread data.

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SOURCES

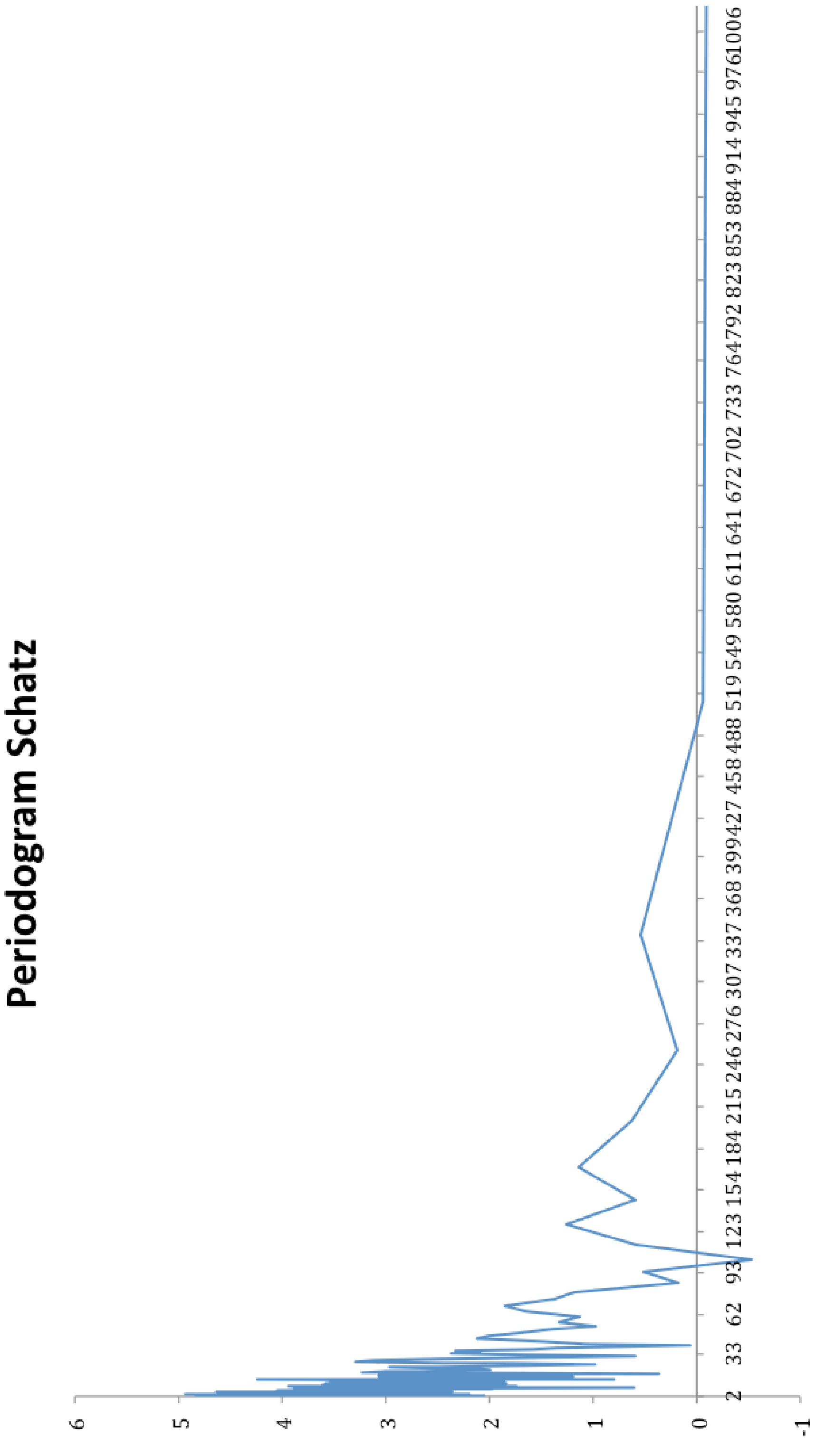
DATA

Bloomberg database

WEB

http://www.cmegroup.com/trading/interest-rates/us-treasury/10-year-us-treasury-note.html (2011-06-30)

APPENDIX



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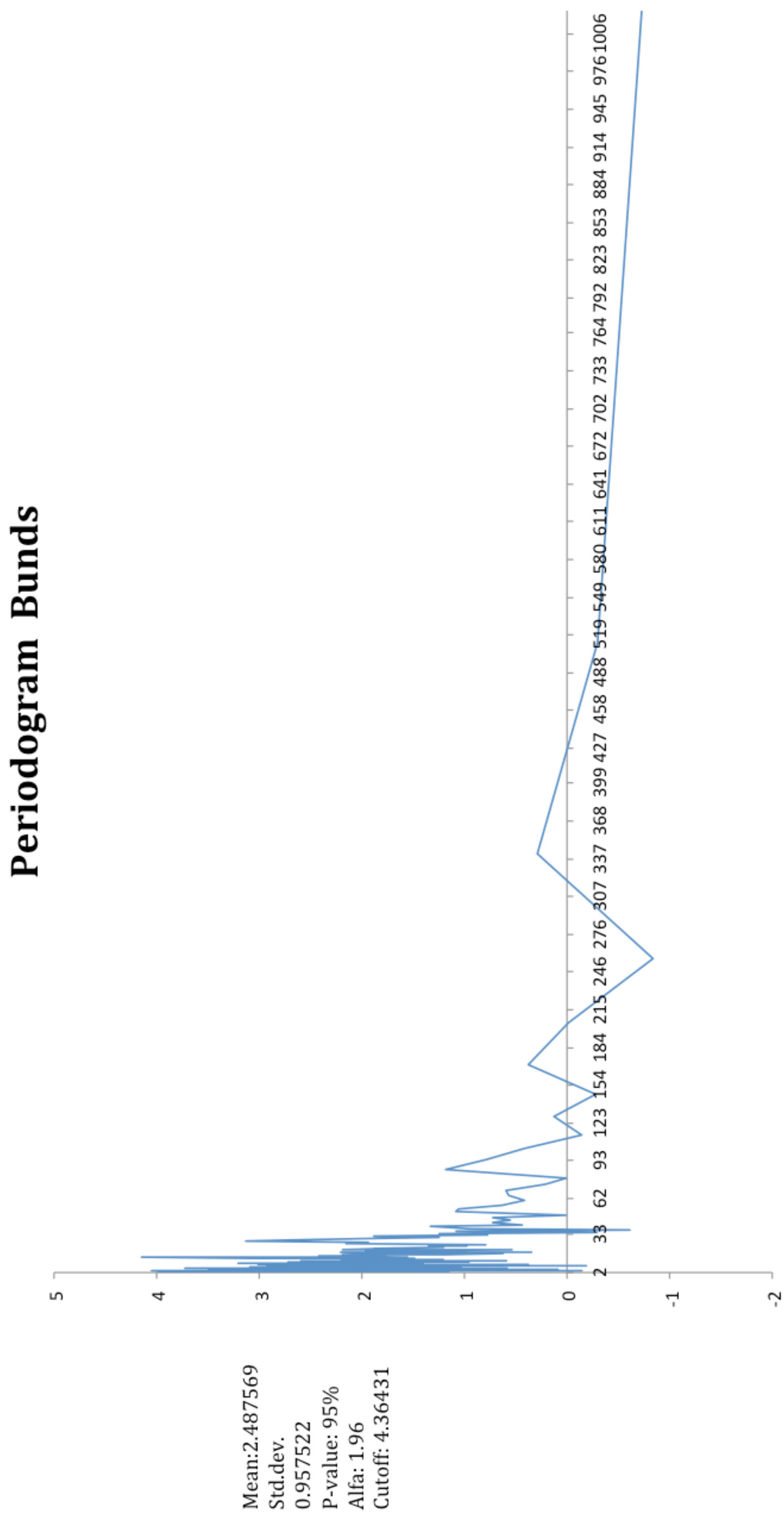
Std. 1.013658
P-value: 95%
Alfa: 1.96
Cutoff:
5.465852


Periodogram Bunds
5
4
3
2
1
0
-1
-2
2 33 62 93 123 154 184 215 246 276 307 337 368 399 427 458 488 519 549 580 611 641 672 702 733 764 792 823 853 884 914 945 976 1006
Mean:2.487569
Std.dev.
0.957522
P-value: 95%
Alfa: 1.96
Cutoff: 4.36431

CORRELOGRAM

Bund

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.420	-0.420	277.18	0.000
		2	0.002	-0.212	277.18	0.000
		3	0.020	-0.087	277.80	0.000
		4	-0.042	-0.086	280.59	0.000
		5	-0.010	-0.081	280.74	0.000
		6	-0.002	-0.065	280.75	0.000
		7	0.030	-0.006	282.20	0.000
		8	-0.082	-0.096	292.75	0.000
		9	-0.014	-0.125	293.04	0.000
		10	-0.002	-0.117	293.05	0.000
		11	0.021	-0.064	293.72	0.000
		12	0.010	-0.034	293.90	0.000
		13	0.039	0.021	296.29	0.000
		14	-0.033	-0.017	298.00	0.000
		15	0.023	0.006	298.82	0.000
		16	-0.010	-0.010	298.99	0.000
		17	0.028	0.021	300.21	0.000
		18	0.002	0.025	300.21	0.000
		19	-0.034	-0.016	302.00	0.000
		20	-0.026	-0.056	303.05	0.000
		21	-0.015	-0.063	303.39	0.000
		22	0.003	-0.051	303.40	0.000
		23	-0.004	-0.047	303.43	0.000
		24	-0.030	-0.093	304.91	0.000
		25	0.005	-0.089	304.94	0.000
		26	0.001	-0.078	304.95	0.000
		27	0.033	-0.030	306.68	0.000
		28	0.083	0.081	317.66	0.000
		29	-0.019	0.068	318.23	0.000
		30	0.025	0.081	319.19	0.000
		31	0.032	0.129	320.78	0.000
		32	-0.046	0.078	324.11	0.000
		33	-0.044	-0.009	327.17	0.000
		34	0.055	0.046	332.09	0.000
		35	-0.045	0.021	335.35	0.000
		36	-0.019	0.013	335.94	0.000

Schatz

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.416	-0.416	282.14	0.000
		2	-0.041	-0.259	284.85	0.000
		3	-0.000	-0.168	284.85	0.000
		4	-0.028	-0.149	286.11	0.000
		5	0.024	-0.089	287.02	0.000
		6	-0.003	-0.063	287.03	0.000
		7	-0.040	-0.098	289.61	0.000
		8	0.004	-0.094	289.63	0.000
		9	-0.023	-0.118	290.50	0.000
		10	0.016	-0.098	290.93	0.000
		11	-0.033	-0.137	292.69	0.000
		12	0.035	-0.094	294.74	0.000
		13	0.026	-0.043	295.85	0.000
		14	-0.031	-0.068	297.40	0.000
		15	-0.000	-0.076	297.40	0.000
		16	0.025	-0.043	298.42	0.000
		17	0.014	-0.011	298.74	0.000
		18	0.010	0.014	298.91	0.000
		19	-0.038	-0.022	301.27	0.000
		20	-0.042	-0.090	304.12	0.000
		21	0.027	-0.075	305.35	0.000
		22	0.006	-0.064	305.41	0.000
		23	-0.005	-0.065	305.45	0.000
		24	-0.035	-0.114	307.47	0.000
		25	0.024	-0.098	308.45	0.000
		26	-0.026	-0.144	309.53	0.000
		27	0.022	-0.141	310.31	0.000
		28	0.069	-0.050	318.15	0.000
		29	0.006	-0.001	318.21	0.000
		30	-0.007	0.013	318.30	0.000
		31	0.006	0.039	318.36	0.000
		32	-0.026	0.028	319.51	0.000
		33	-0.019	-0.010	320.10	0.000
		34	0.008	-0.014	320.20	0.000
		35	0.022	0.034	320.98	0.000
		36	-0.034	0.027	322.87	0.000

US

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.453	-0.453	478.43	0.000
		2	-0.007	-0.268	478.54	0.000
		3	-0.015	-0.190	479.05	0.000
		4	0.018	-0.112	479.80	0.000
		5	-0.004	-0.073	479.83	0.000
		6	0.004	-0.041	479.88	0.000
		7	0.006	-0.013	479.96	0.000
		8	-0.030	-0.043	482.03	0.000
		9	0.015	-0.029	482.56	0.000
		10	0.017	0.004	483.24	0.000
		11	-0.015	-0.005	483.79	0.000
		12	0.019	0.021	484.60	0.000
		13	-0.027	-0.010	486.35	0.000
		14	0.029	0.020	488.37	0.000
		15	-0.023	-0.003	489.63	0.000
		16	-0.008	-0.026	489.78	0.000
		17	-0.011	-0.047	490.07	0.000
		18	0.002	-0.050	490.08	0.000
		19	-0.000	-0.047	490.08	0.000
		20	-0.019	-0.067	490.92	0.000
		21	0.011	-0.054	491.21	0.000
		22	-0.010	-0.057	491.46	0.000
		23	0.003	-0.049	491.49	0.000
		24	-0.015	-0.063	492.00	0.000
		25	0.024	-0.027	493.33	0.000
		26	-0.042	-0.071	497.56	0.000
		27	0.022	-0.050	498.65	0.000
		28	-0.017	-0.065	499.33	0.000
		29	-0.023	-0.100	500.54	0.000
		30	0.029	-0.066	502.49	0.000
		31	-0.020	-0.080	503.44	0.000
		32	-0.016	-0.109	504.05	0.000
		33	0.004	-0.113	504.09	0.000
		34	0.028	-0.075	505.96	0.000
		35	-0.013	-0.078	506.38	0.000
		36	0.013	-0.058	506.77	0.000

GPH-test

Future	Coefficient	Std. Error	t-statistic	Prob.
Bund	C(1): 2.485684	0.042351	58.69197	0.0000
	C(2): 0.007171	0.023101	0.310402	0.7564
Schatz	C(1): -1.180172	0.0053949	-21.87583	0.0000
	C(2): 0.006332	0.028421	0.222794	0.8238
US	C(1): 2.779227	0.030182	92.08148	0.0000
	C(2): -0.010647	0.017109	-0.622313	0.5339

Phillips-Perron-test

Future	Series	Adj. t-statistic	Prob.
Bund	Integrated spread:	-138.7716	0.0000
	Trading activity:	-30.48907	0.0001
Schatz	Integrated spread:	-390.9659	0.0000
	Trading activity:	-31.51051	0.0000
US	Integrated spread:	-251.6166	0.0001
	Trading activity:	-39.72811	0.0000

Breusch-Pagan-Godfrey-test

Future	F-statistic	Probability
Bund	0.011973	Prob.F(1.1567) 0.9129
Schatz	0.413672	Prob.F(1.1623) 0.5202
US	6.976483	Prob. F(1.2323) 0.0083

Breusch-Godfrey-test

Future	F-statistic	Probability
Bund	25.86790	Prob.F(2.1563)0.0000
Schatz	7.187367	Prob.F(2.1619) 0.0008
US	1.014797	Prob.F(2.2321)0.3626

Output

Future	Coefficient	Std. Error	t-statistic	Prob.	Adjusted R-squared
Bund	-0.266148	0.005453	-48.80912	0.0000	0.739365
Schatz	-0.205074	0.004946	-41.45879	0.0000	0.691097
US	-0.330456	0.007059	-46.87728	0.0000	0.733414

Wald-test

Future	t-statistic	Degrees of freedom	Probability
Bund	11.70989	1565	0.0000
Schatz	25.94341	1621	0.0000
US	-0.080031	2323	0.9362